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Electromagnetic corrections in the anomaly sector and the π^0 lifetime

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Abstract. The framework for computing radiative corrections in ChPT in the anomaly sector is set up. Application to the lifetime of the π^0 and the possibility to extract information on $m_d - m_u$ from precision data is discussed.

PACS. 11.30.Rd Chiral symmetries – 12.15.Ff Quark and lepton masses and mixing – 12.39.Fe Chiral Lagrangians

1 Introduction

In order to determine the masses of the light quarks from experimental data, it is necessary to consider isospin-breaking phenomena which are induced in part by the quark mass difference $m_u - m_d$. The second source of isospin violation, radiative effects, must then also be taken into account. In this paper I will present the formalism which allows one to take into account radiative corrections in ChPT in the anomaly sector, extending the work of Urech [1] who has shown how to do so in the ordinary sector (there as still some open theoretical questions in this field, see the contribution by A. Rusetsky [2] to this conference). As an application I will discuss the lifetime of the neutral pion in ChPT at next-to-leading order. In this problem, the corrections to the current algebra result are dominated by isospin breaking, and the precision of the experimental measurement now underway at CEBAF (Primex experiment [3]) allows one to probe these corrections. I will show that in this problem, it is possible to quantitatively estimate the contributions of electromagnetic origin. This presentation summarizes the work done in collaboration with B. Ananthanarayan which is described in more detail in ref. [4]

2 Formalism

Introducing the photon as a dynamical field in the chiral expansion induces new terms in the Lagrangian which parametrize the high-energy part of the photon loops. One must then solve the problem of classifying a minimal set of independent terms at each chiral order and the related problem of determining the divergence structure of the associated chiral coupling constants. In ref. [4] we have considered this problem for the case of the $SU(2)$ chiral expansion. Because of the limited space I will only mention the results here. At leading order (which is p^4) one has the WZW [5,6] action for which Kaiser [7] has recently derived a particularly simple form,

$$\mathcal{L}^W_{p^4} = \kappa\, \epsilon^{\mu\nu\alpha\beta}\Big(\langle U^\dagger r_\mu U l_\nu - r_\mu l_\nu + i\Sigma_\mu (U^\dagger r_\nu U + l_\nu)\rangle \langle v_{\alpha\beta}\rangle + \frac{2}{3}\langle \Sigma_\mu \Sigma_\nu \Sigma_\alpha\rangle\langle v_\beta\rangle\Big) \quad (1)$$

with $\kappa = -N_c/32\pi^2$. We will consider terms of order e^2p^4: according to the non-renormalization theorem [8] they must be non-anomalous and therefore may be constructed from standard chiral building blocks (see, *e.g.*, ref. [9]). We found that there are exactly eight independent terms at this order which may be chosen as follows:

$$\begin{aligned}\mathcal{L}^W_{e^2p^4} &= \epsilon^{\mu\nu\alpha\beta} \\ &\times\Big\{ k^W_1 i\langle Q_+\rangle\langle Q_-u_\mu\rangle\langle u_\nu u_\alpha u_\beta\rangle + k^W_2\langle [Q_+,Q_-]u_\alpha u_\beta\rangle\langle f_{+\mu\nu}\rangle \\ &+ k^W_3\langle Q_+\rangle\langle [Q_-, f_{+\mu\nu}]u_\alpha u_\beta\rangle + k^W_4\langle Q_+^2 u_\alpha\rangle\langle f_{-\mu\nu}u_\beta\rangle \\ &+ k^W_5 i\langle [Q_+, f_{+\mu\nu}]Q_-\rangle\langle f_{+\alpha\beta}\rangle + k^W_6 i\langle Q_+\rangle\langle [Q_+, f_{+\mu\nu}]f_{-\alpha\beta}\rangle \\ &+ k^W_7 i\langle [Q_{+\mu}, Q_+]u_\nu\rangle\langle f_{+\alpha\beta}\rangle + k^W_8 i\langle [Q_{-\mu}, Q_-]u_\nu\rangle\langle f_{+\alpha\beta}\rangle\Big\}.\end{aligned} \quad (2)$$

As usual in ChPT, the couplings k^W_i are divergent. The divergence is generated from one-loop diagrams containing one vertex from the Lagrangian (1). As in the non-anomalous case, there are both diagrams with explicit photons and diagrams with pions in which the EM effects are induced from the π^+-π^0 mass difference. In the

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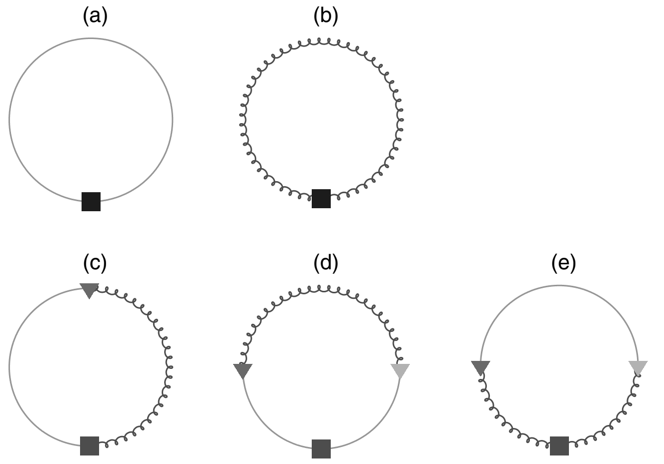

Fig. 1. One-particle-irreducible graphs which contribute to the $O(p^6)$ and the $O(e^2p^4)$ divergences in the anomalous sector: a triangle denotes an $O(p^2)$ vertex a box denotes an $O(p^4)$ vertex. The solid line represents the (full) pion propagator and the curly line the full photon propagator.

present case, because the chiral order is p^6, one must only be careful that there are contributions which are one particle reducible. The set of irreducible graphs that one must compute is shown in fig. 1. Putting all these contributions together and defining the relation between bare and renormalized couplings as follows:

$$k^W_{i,\mathrm{bare}} = (c\mu)^{d-4}\left(\frac{1}{16\pi^2(d-4)}\beta_i + k^W_{i,r}(\mu)\right)\ , \tag{3}$$

we find that the coefficients β_i have the following values in the Feynman gauge:

$$\beta_1 = \beta_6 = \beta_8 = 0 \tag{4}$$

and

$$\begin{aligned}\beta_2 &= \left(\tfrac{1}{8} - \tfrac{1}{6}\tfrac{C}{F^4}\right)\kappa, \qquad \beta_3 = -\tfrac{1}{8}\kappa,\ \ \beta_4 = -\tfrac{1}{4}\kappa,\\ \beta_5 &= \left(-\tfrac{1}{16} + \tfrac{1}{6}\tfrac{C}{F^4}\right)\kappa,\ \ \beta_7 = \tfrac{1}{4}\kappa\ .\end{aligned} \tag{5}$$

C is the EM chiral parameter which induces π^+-π^0 mass splitting at leading order: $m^2_{\pi^+} - m^2_{\pi^0} = 2e^2C/F^2_\pi$. The divergence structure for other gauges is also discussed in ref. [4].

3 π^0 lifetime at NLO

We would like here to collect in a complete way the contributions to the π^0 lifetime beyond the current algebra prediction including, in particular, the electromagnetic contributions. One has in mind to match from the theoretical side a precision of the order of 1% which is the typical precision expected from the PrimEx experiment [3]. This involves three steps:

Step 1: One must be able to determine F_π, defined as the axial pion coupling *in the absence of QED* to high precision, as it is the basic coupling entering the chiral expansion.

Step 2: One must perform a ChPT calculation up to one loop (including the photon) and include the tree level contributions of the relevant chiral order (*i.e.* up to order six here, since the leading order is four). In this manner, one obtains an expression for the decay amplitude in terms of a (numerous) set of chiral couplings.

Step 3: One must evaluate the chiral couplings involved. The real interest of this enterprise is that this turns out to be actually possible. Let us now go through these steps in turn.

The problem involved in step 1 was solved by Marciano and Sirlin [10]. Making use of some improvements in the modelling of form factors performed in ref. [11] on obtains the value $F_\pi = 92.16 \pm 0.11$ MeV. We note that the precision matches the one that we are after and that, as defined, this coupling is exactly the same for charged or for neutral pions at this level of accuracy. Turning to step 2 now, we choose to perform the calculation in the framework of the $SU(3)$ expansion for, otherwise, we would not be able to successfully go through step 3. At leading order, the amplitude is determined from the WZW Lagrangian and also picks up a contribution from π^0-η mixing:

$$A_{p^4} = \frac{\alpha}{\pi F_\pi}\left(1 + \frac{m_d - m_u}{4(m_s - m)}\right), \quad m = \frac{1}{2}(m_u + m_d)\,. \tag{6}$$

At NLO let us consider the one-particle *irreducible* contributions (the parametrization of the chiral field and of the fluctuations being the same as in ref. [12]) first: there are no photon loop contribution and the sum of pseudo-scalar loops vanish [13,14]. As for the tree level contributions, there are none coming from $\mathcal{L}^W_{e^2p^4}$ and the only contributions are from $\mathcal{L}^W_{p^6}$. Using the notations of ref. [15], they involve the two coupling-constants C^W_7, C^W_8. Next come the one-particle-*reducible* contributions. These can be encoded as a matrix of wave function renormalization and mixing. π^0-η mixing whose at the leading oder is described by one mixing parameter ϵ now involves two mixing parameters ϵ_1, ϵ_2, whose expressions can be found in ref. [12]. Only ϵ_2 occurs in our problem:

$$\begin{aligned}\epsilon_2 = {}& \frac{\sqrt{3}(m_d - m_u)}{4(m_s - m)}\\ & \times\left\{1 - \frac{32(m^2_K - m^2_\pi)}{F^2_0}(3L_7 + L_8) + \chi_{\mathrm{logs}}\right\}\,.\end{aligned} \tag{7}$$

At this level, electromagnetic contributions are present, which involve the chiral couplings K_i introduced by Urech. They can be collected in two terms:

$$\begin{aligned}\delta_2 = {}& \frac{\sqrt{3}\,e^2 m}{m_s - m}\Big(- (2K_3 - K_4) + \frac{2}{3}(K_5 + K_6)\\ & -\frac{2}{3}(K_9 + K_{10}) - \frac{2C}{F^4_0}\tilde{\nu}_K\Big)\end{aligned} \tag{8}$$

and

$$\delta_\pi = e^2 \Big\{ -\frac{4}{3}(K_1+K_2) + (2K_3 - K_4) - \frac{10}{9}(K_5+K_6) + \frac{C}{F_0^4}(4\tilde{\nu}_\pi + \tilde{\nu}_K) \Big\} . \qquad (9)$$

From this, we can write the $\pi^0 \to 2\gamma$ amplitude including all the contributions which are of chiral order p^6 or e^2p^4, *i.e.* linear in the quark masses m_u, m_d and the electric charge e^2

$$A_{\pi 2\gamma} = \frac{\alpha}{\pi F_\pi} \Big\{ 1 + \frac{\epsilon_2}{\sqrt{3}} + \frac{\delta_2}{\sqrt{3}} + \delta_\pi + \left(1 - \frac{m_d - m_u}{m} \frac{4r-5}{4(r-1)}\right) T_1 - \frac{3(m_d - m_u)}{4m} T_1' \Big\} , \qquad (10)$$

where r is the mass ratio m_s/m and

$$T_1 = -\frac{256\pi^2}{3} m_\pi^2 C_7^W , \quad T_1' = -\frac{1024\pi^2}{3} m_\pi^2 C_8^W . \qquad (11)$$

We note that all the contributions are isospin violating except for one term proportional to C_7^W. Let us now discuss the determination of the various couplings. Those appearing in ϵ_2: L_7 and L_8 have been discussed in ref. [12]. The couplings C_7^W, C_8^W can be estimated from resonance saturation models using methods proposed in ref. [16]. Equivalently, one can express both of them as convergent sum rules and then estimate the integrals using resonance saturation. C_8^W encodes the influence of π^0-η' mixing, while C_7^W has an analogous interpretation in terms of π^0-$\pi^0(1300)$ mixing. The experimentally observed suppression of the $\pi(1300)$ electromagnetic decay then leads to the fact that C_7^W is strongly suppressed compared to C_8^W. The latter parameter can then be determined more precisely using ChPT and input from η electromagnetic decay. There remains to estimate the combination of Urech couplings K_i which appear in δ_π and δ_2. This again, can be done using resonance saturation models. These models must be constrained by chiral symmetry and also by proper QCD asymptotic conditions. Assuming dominance of the vector meson resonances here a unique answer is found for the two combinations

$$\frac{4}{3}(K_1+K_2) - (2K_3 - K_4) + \frac{10}{9}(K_5+K_6) = \frac{5g_1^2}{48\pi^2} ,$$
$$-\frac{2}{3}(2K_3 - K_4) + \frac{4}{9}(K_5+K_6) = \frac{g_1^2}{24\pi^2} , \qquad (12)$$

in terms of a vector meson coupling constant g_1 which can be precisely determined from the $\omega \to \pi\gamma$ decay width. Putting these results together one can express the pion decay amplitude as follows in terms of $m_u - m_d$:

$$A_{\pi 2\gamma} = \frac{\alpha}{\pi F_\pi} \Big\{ 1 - 0.34 10^{-2} + \frac{m_d - m_u}{m_s - m}(0.93 \pm 0.12) \pm 0.14 10^{-2} \Big\} . \qquad (13)$$

The electromagnetic contributions are found to decrease the decay width by approximately 0.6%. The order of magnitude of the quark mass ratio involved is $1/R \equiv (m_d - m_u)/(m_s - m) \simeq 1/43$ (*e.g.* [17]). This generates a contribution which increases the pion decay width by approximatively 5%. Such a contribution can be probed at the qualitative level by the PrimEx experiment. Assuming the above value for R, the theoretical prediction for the pion decay width would be

$$\Gamma_{\pi 2\gamma} = 8.06 \pm 0.02 \pm 0.06 \text{ eV}, \qquad (14)$$

where the first error originates from the determination of F_π and second collects the incertainties in the values of the coupling constants. A very similar result was obtained recently in ref. [18] using an expansion scheme which combines the chiral and the large-N_c expansions. One reason for the smallness of the NLO corrections is the suppression of the contributions which are not isospin breaking (other than those which can be absorbed into F_π). Such isospin-conserving contributions could be present at NNLO and it would be useful, in the future, to try to estimate their order of magnitudes.

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